

DMC FILE COPY

UNCLASSIFIED

①

## REPORT DOCUMENTATION PAGE

AD-A201 363

1b. RESTRICTIVE MARKINGS

3. DISTRIBUTION/AVAILABILITY OF REPORT

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

FJSRL-JR-88-0012

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION

Frank J. Seiler Research Lab

6b. OFFICE SYMBOL

(If applicable)

FJSRL/NH

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS (City, State and ZIP Code)

USAF Academy

Colorado Springs, CO 80840-6528

7b. ADDRESS (City, State and ZIP Code)

8a. NAME OF FUNDING/SPONSORING ORGANIZATION

8b. OFFICE SYMBOL

(If applicable)

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State and ZIP Code)

10. SOURCE OF FUNDING NOS.

PROGRAM  
ELEMENT NO.PROJECT  
NO.TASK  
NO.WORK UNIT  
NO.11. TITLE (Include Security Classification) Integral Solution for  
Diffraction Problems Involving Conducting Surfaces  
with Complex Geometries. III. Application to

12. PERSONAL AUTHOR(S) Paraboloidal Mirrors (U)

Mohamed F. El-Hewie

13a. TYPE OF REPORT

Journal Publication

13b. TIME COVERED

FROM \_\_\_\_\_ TO \_\_\_\_\_

14. DATE OF REPORT (Yr., Mo., Day)

September 1988

15. PAGE COUNT

6

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

paraboloidal mirrors; diffraction; raytracing, Prop. to 1/13

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The complex ray-tracing theory developed earlier (J. Opt. Soc. Am. A 5, 200 (1988)) is applied to a study of the structure of the electromagnetic focal field spectrum of paraboloidal mirrors. This problem was treated recently by Barakat (Appl. Opt. 26, 3790 (1987)), who used the Gaussian vectorial diffraction method. In this paper a more general solution to the problem is presented that uses the Stratton-Chu-Silver integral. The reflecting kernel then explicitly incorporates the surface function, the surface physical parameters and their dependence on the state of polarization of the incident radiation, and an unrestricted aberration function. Numerical results are presented for mirrors made of four different metals, and their use in optimizing the mirror is demonstrated. Key words:

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT

UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☒ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION

UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL

Mohamed F. El-Hewie

22b. TELEPHONE NUMBER  
(Include Area Code)

719-472-3122

22c. OFFICE SYMBOL

FJSRL/NH

Reprinted from Journal of the Optical Society of America A, Vol. 5, page 1444, September 1988  
Copyright © 1988 by the Optical Society of America and reprinted by permission of the copyright owner.

# Integral solution for diffraction problems involving conducting surfaces with complex geometries. III. Application to paraboloidal mirrors

Mohamed F. El-Hewie

Frank J. Seiler Research Laboratory, U.S. Air Force Academy, Colorado Springs, Colorado 80840-6528

Received October 26, 1987; accepted May 6, 1988

The complex ray-tracing theory developed earlier [J. Opt. Soc. Am. A 5, 200 (1988)] is applied to a study of the structure of the electromagnetic focal field spectrum of paraboloidal mirrors. This problem was treated recently by Barakat [Appl. Opt. 26, 3790 (1987)], who used the Gaussian vectorial diffraction method. In this paper a more general solution to the problem is presented that uses the Stratton-Chu-Silver integral. The reflecting kernel then explicitly incorporates the surface function, the surface physical parameters and their dependence on the state of polarization of the incident radiation, and an unrestricted aberration function. Numerical results are presented for mirrors made of four different metals, and their use in optimizing the mirror design is demonstrated.

## INTRODUCTION

In part I of this series a complex ray-tracing theory was developed for the solution of diffraction problems of complex conducting surfaces.<sup>1</sup> In part II the theory was applied to the ellipsoidal scatterers.<sup>2</sup> Recently Barakat<sup>3</sup> studied the structure of the focal spectrum of a paraboloidal mirror, assuming that the mirror is absolutely reflecting, the incident rays are symmetrical and meridional, and the system is aberration free. In what follows it is shown that these assumptions can easily be eliminated without adding much complexity to the method of solution.

First, if the mirror is metallic, the real refractive index varies with the angle of incidence. For example, at radiation wavelengths  $\lambda = 0.1 \mu\text{m}$  and  $\lambda = 1.315 \mu\text{m}$  the real refractive index of aluminum is, respectively, 14.64 and 1.00 at normal incidence and 14.66 and 1.06 at grazing incidence.<sup>2</sup> Thus the reflectivity and absorptivity vary with the angle of ray incidence upon a metallic surface, and both vary with the state of wave polarization for both metallic and dielectric surfaces.<sup>2,4</sup> Therefore the conservation of energy in a ray tube [Ref. 5, Eq. (2.11)] and that carried out by Barakat [Ref. 3, Eq. (14)] should yield the polarization and angular dependence of the surface reflectivity in the aperture function,  $q(\theta)$ . Second, asymmetrical reflective focusing is of more interest in practice. Inertial confinement fusion devices, in which the laser beams strike the focusing mirrors obliquely, and astrophysical observatories, for which unobstructed images are desired, are just two practical examples. Because in Barakat's approach the Kirchhoff integral is evaluated on a Gaussian sphere centered at the Gaussian focus, asymmetrical focusing can hardly be analyzed by his approach. Third, the intended inclusion of some aberration orders, third-order spherical and third-order coma, can easily be replaced by the whole aberration function by using the Stratton-Chu-Silver integral in place of the Kirchhoff integral.

In this paper a general treatment is presented for the focusing of a plane wave and a Gaussian laser beam by an aluminum-coated paraboloidal mirror. Three other metal

coatings are studied for comparison with aluminum. The surface geometrical and physical parameters, the state of radiation polarization, and the whole wave-front-aberration function are explicitly incorporated into the reflecting kernel.

## MATHEMATICAL FORMULATION

The equations of parts I and II, respectively, cited here will be preceded by the numerals I and II. Given a plane wave with a propagation constant  $k = 2\pi/\lambda$  and a propagation unit vector  $\hat{n}_i$ , Eq. (I.10a) gives

$$q(Z, \sigma, \omega) = -(\xi\Psi - \zeta\Lambda)/k + [\sin\theta_i(Z_x \cos\varphi_i + Z_y \sin\varphi_i) - \cos\theta_i]/(1 + Z_x^2 + Z_y^2)^{1/2}. \quad (1)$$

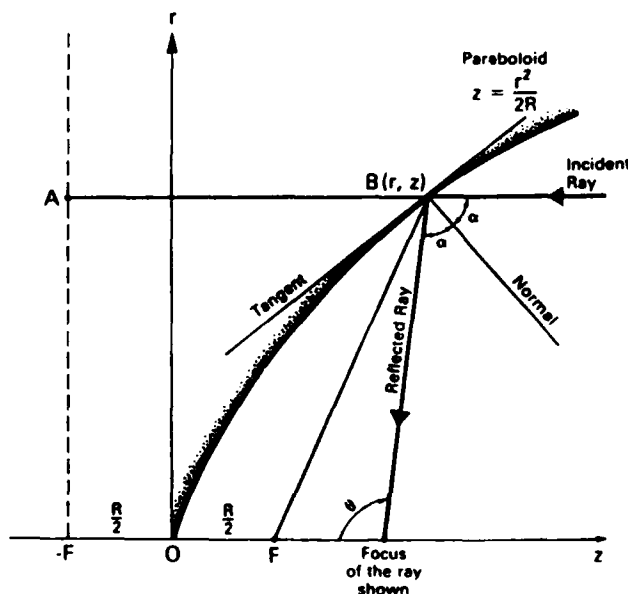


Fig. 1. Geometry of the problem.

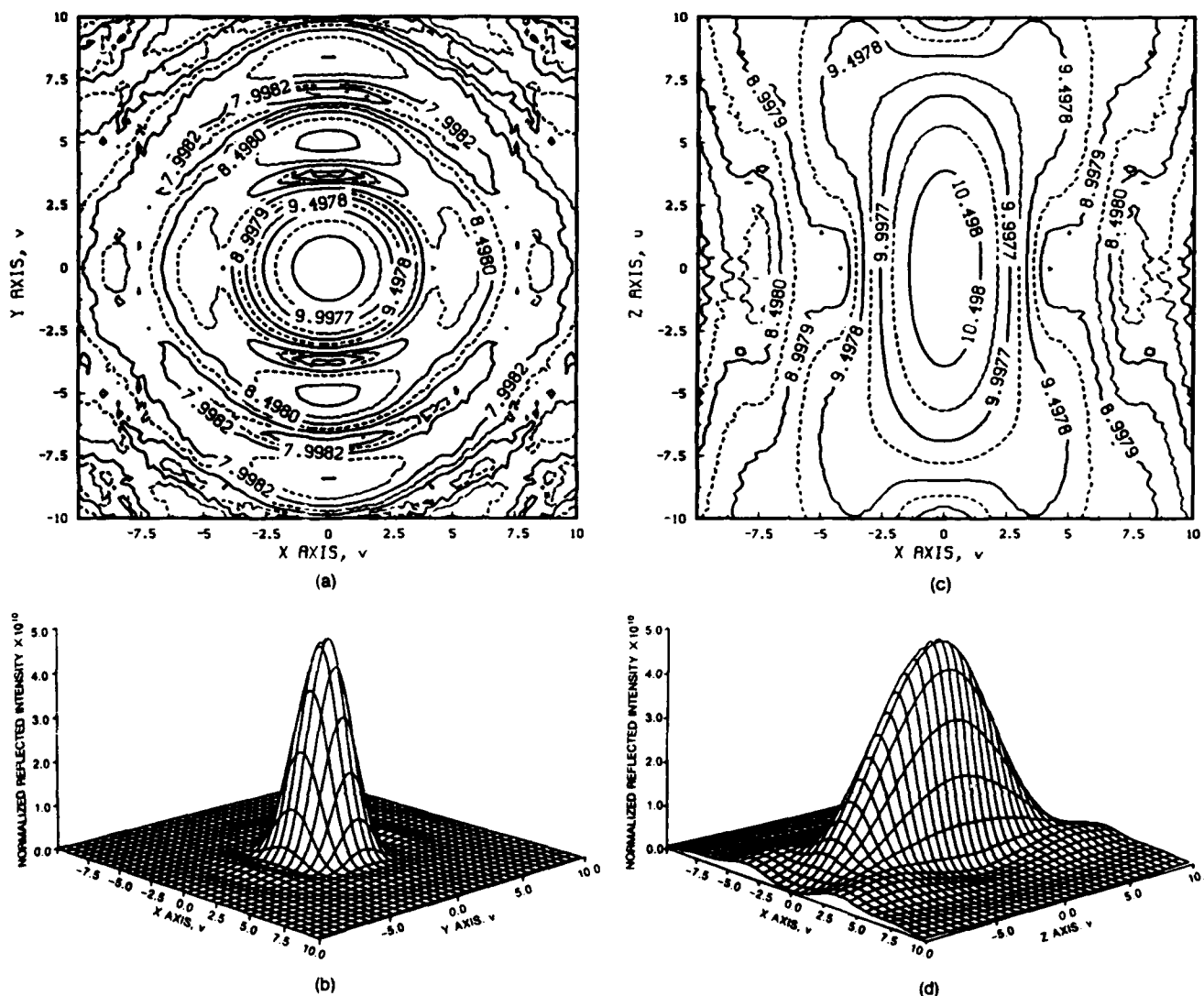


Fig. 2. Results of Ref. 3, Figs. 10-12, reproduced, using our computational code, with a perfectly reflecting paraboloidal mirror with a focal length of  $R/2 = 60$  cm and a height of  $d = 10$  cm. The following notation is used:  $u = kx \sin \alpha$  on the  $x$  axis and  $v = ky \sin \alpha$  on the  $y$  axis,  $\sin \alpha = (2Rd)^{1/2}/(R/2)$ , and  $u = kz \sin \alpha$ . (a) Contours of the reflected energy in the  $xy$  plane at the focus,  $z = -R/2$  (the contour labels are  $\log_{10}$  of the reflected energy normalized to the maximum incident energy). (b) Surface plot of the energy in the  $xy$  focal plane. (c), (d) Correspond to (a) and (b), respectively, but in the  $xz$  plane with  $z = 0$  in these figures corresponding to the focal point in Fig. 1.

Equations (1.8) then give  $\Psi(Z, \sigma, \omega)$  and  $\Lambda(Z, \sigma, \omega)$  in terms of the local angle of incidence  $\theta_{in}$ , defined below by Eq. (6).

The incident electromagnetic fields have field strength distribution  $E_i(Z)$  and aberration function  $u(Z)$  given as follows.

For a plane wave,

$$E_i(Z) = E_0 \exp(-ikw) \quad (2a)$$

and

$$u = 0. \quad (2b)$$

For a Gaussian beam,<sup>6</sup>

$$E_i(Z) = E_0(w_0/w) \exp[-(r/w)^2 - ik(w - |u|)] \quad (2c)$$

and

$$u = [(1/k) \tan^{-1}(2w/kw_0^2) - r^2/2\rho] \hat{n}_i. \quad (2d)$$

Here,  $w$  is the axial distance along the laser beam, measured from the beam waist;  $w_0$  is the beam-waist parameter;  $w_i$  is a beam parameter defined by  $w_i^2 = w_0^2[1 + (2w/kw_0^2)^2]$ ;  $r$  is the radial distance in the beam frame of coordinates; and  $\rho$  is the wave-front curvature at the distance  $w$  and is given by  $\rho = w + (kw_0^2/2)^2/w$ . Thus the  $F$ 's in Eqs. (1.14) are determined as follows:

For a plane wave,

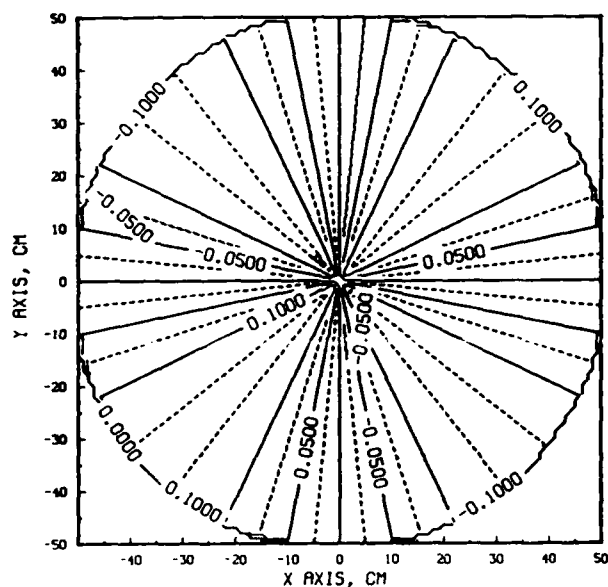
$$F_1 = \sin \theta_i \cos \varphi_i, \quad (3a)$$

$$F_2 = \sin \theta_i \sin \varphi_i, \quad (3b)$$

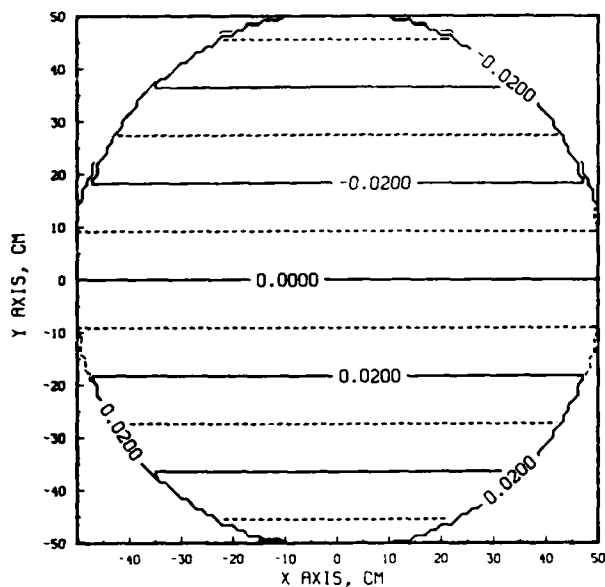
and

$$F_3 = \cos \theta_i. \quad (3c)$$

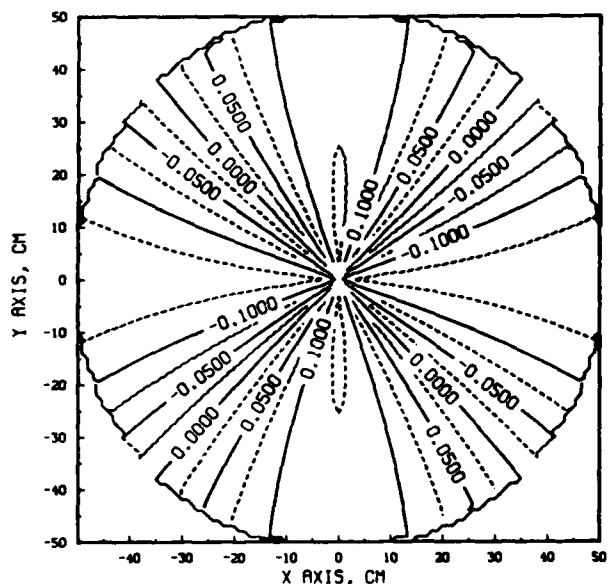
For a Gaussian laser beam,



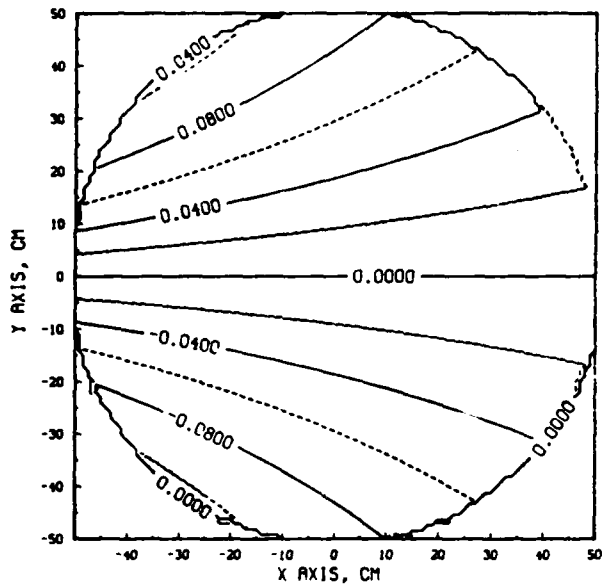
(a)



(c)



(b)



(d)

$$F_1 = \sin \theta_i \cos \varphi_i [1 + (1/k) \tan^{-1}(2w/kw_0^2) - r^2/2(w - \rho)], \quad (3d)$$

$$F_2 = \sin \theta_i \sin \varphi_i [1 + (1/k) \tan^{-1}(2w/kw_0^2) - r^2/2(w - \rho)], \quad (3e)$$

and

$$F_3 = \cos \theta_i [1 + (1/k) \tan^{-1}(2w/kw_0^2) - r^2/2(w - \rho)]. \quad (3f)$$

Now, the coordinate function of a paraboloidal mirror (Fig. 1) can be written in the beam frame ( $Z = -z$ ) as

$$Z = -(x^2 + y^2)/2R \quad \text{for } -d \leq Z \leq 0, \quad (4)$$

where  $R/2$  is the focal length of the paraboloidal mirror. The origin of the frame of coordinates is taken at the vertex of the paraboloid, and the  $z$  axis is its axis of revolution. The spatial derivatives of  $Z$  are

$$Z_x = -x/R \quad (5a)$$

and

$$Z_y = -y/R. \quad (5b)$$

The local angle of incidence is defined by

$$\cos \theta_{in} = [-\sin \theta_i (Z_x \cos \varphi_i + Z_y \sin \varphi_i) + \cos \theta_i] / (1 + Z_x^2 + Z_y^2)^{1/2}. \quad (6)$$

Substituting the  $F$ 's,  $Z_x$ , and  $Z_y$  from Eqs. (3) and (5) into Eqs. (1) and (1.15) gives the real part of the complex refractive index,  $\nu$ , for the surface  $Z(x, y)$  as a function in  $Z$ ,  $Z_x$ ,  $Z_y$ ,  $\sigma$ , and  $u$ .

The Fresnel coefficients of reflection and transmission, written in terms of  $\nu$ , are then determined in a form similar to Eqs. (11.9). Then Eqs. (1.19) give the various field vectors  $e_r^i$ ,  $h_r^i$ ,  $e_p^i$ , and  $h_p^i$ , and Eqs. (1.21) and (1.22) give the local

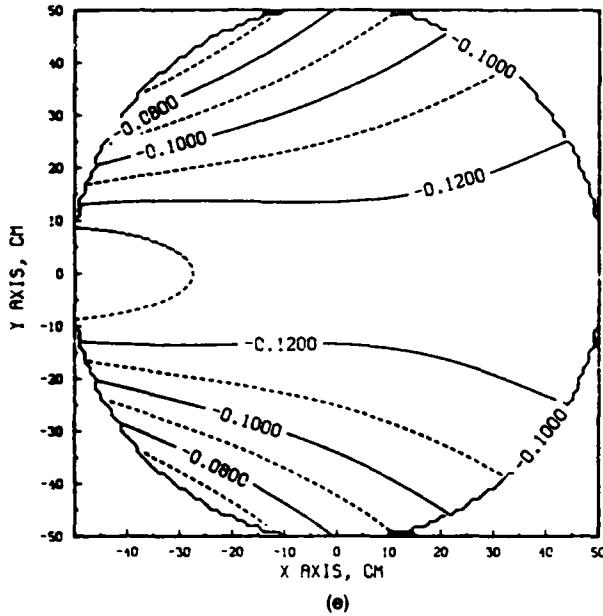


Fig. 3. The  $S$ 's [surface currents at an aluminum paraboloidal mirror surface, Eqs. (10)]. (a)–(c) Are for  $\theta_i = 0$  incidence, and (d), (e) are for  $45^\circ$  incidence in the  $\varphi_i = 0$  plane. (a), (d),  $S_1$ . (b), (e)  $S_2$ . (c)  $S_3$ .

currents on the surface  $Z$ ,  $J_s^i$  and  $J_p^i$ . Thus both absorption and reflecting parameters at the surface are fully determined.

To simplify numerical calculations, only the case of a plane wave incident at  $\theta_i = 0$ ,  $\varphi_i = 0$ , and  $\alpha = 0$  will be considered. For this case, Eqs. (I.18) give

$$\hat{t} = [iZ_y \cos \theta_i - j(\sin \theta_i + Z_x \cos \theta_i) - kZ_y \sin \theta_i]/C \sin \theta_{in}, \quad (7a)$$

$$\hat{p} = [i[\sin \theta_i(Z_x^2 + 1) + Z_x \cos \theta_i] + j[-Z_x Z_y \sin \theta_i + Z_y \cos \theta_i] + k[\cos \theta_i(Z_x^2 + Z_y^2) + Z_x \sin \theta_i]]/C^2 \sin \theta_{in}, \quad (7b)$$

$$\cos \theta_{in} = (\cos \theta_i - Z_x \sin \theta_i)/(1 + Z_x^2 + Z_y^2)^{1/2}, \quad (7c)$$

$$\hat{a} \cdot \hat{t} = -Z_y/C \sin \theta_{in}, \quad (7d)$$

$$\hat{a} \cdot \hat{h} = (Z_x \cos \theta_i + \sin \theta_i)/C, \quad (7e)$$

and

$$\hat{a} \cdot \hat{p} = [Z_x(2 \sin^2 \theta_i + Z_x \sin \theta_i \cos \theta_i - 1) - \sin \theta_i \cos \theta_i]/C^2 \sin \theta_{in}. \quad (7f)$$

Thus all the field components in the incident partial electromagnetic waves at the surface  $Z(x, y)$  are also fully determined.

At a point  $P(x_p, y_p, z_p)$  external to the surface  $Z(x, y)$ , the reflected field is given by Eq. (I.30). The unit vectors  $\hat{h}$  and  $\hat{h}_p$  are given by

$$\hat{h} = (-Z_x \hat{i} - Z_y \hat{j} + \hat{k})/(1 + Z_x^2 + Z_y^2)^{1/2} \quad (8a)$$

and

$$\hat{h}_p = [(x - x_p)\hat{i} + (y - y_p)\hat{j} + (Z - z_p)\hat{k}]/D, \quad (8b)$$

where  $D = [(x - x_p)^2 + (y - y_p)^2 + (Z - z_p)^2]^{1/2}$ .

Substituting the  $e$ 's and  $h$ 's from Eqs. (I.19) into Eqs.

(I.31) and (I.32), we get expressions similar to Eqs. (II.17) for the surface currents as

$$\hat{n} \times e_s = (E_i/C)(S_1 \hat{i} + S_2 \hat{j} + S_3 \hat{k}) \quad (9a)$$

and

$$\hat{n} \times h_s = (E_i/\eta C)(T_1 \hat{i} + T_2 \hat{j} + T_3 \hat{k}). \quad (9b)$$

The  $S$ 's and the  $T$ 's (the dimensionless parameters that account for the physical and geometrical properties of the surface, the degree of polarization, and the radiation wavelength) are now defined as

$$S_1 = W(-Z_y L_3 - L_2) + \cos \theta_{in}(1 - R_p)M_1, \quad (10a)$$

$$S_2 = W(Z_x L_3 + L_1) + \cos \theta_{in}(1 - R_p)M_2, \quad (10b)$$

$$S_3 = W(-Z_x L_2 + Z_y L_1) + \cos \theta_{in}(1 - R_p)M_3, \quad (10c)$$

$$T_1 = [(1 + R_p)/C](-Z_x M_3 - M_2) - UL_1, \quad (10d)$$

$$T_2 = [(1 + R_p)/C](Z_x M_1 + M_1) - UL_2, \quad (10e)$$

and

$$T_3 = [(1 + R_p)/C](-Z_x M_2 + Z_y M_1) - UL_3. \quad (10f)$$

By executing the vectorial multiplication in Eqs. (I.31) and (I.32), we get

$$W = \hat{a} \cdot \hat{t}(1 + R_p)/C \sin \theta_{in}, \quad (11a)$$

$$U = \hat{a} \cdot \hat{t}(1 - R_p)\cos \theta_{in}/\sin \theta_{in}. \quad (11b)$$

From the expression of  $e_s^i$ , Eq. (I.19a), we get

$$L_1 = Z_y \cos \theta_i, \quad (11c)$$

$$L_2 = -(\sin \theta_i + Z_x \cos \theta_i), \quad (11d)$$

and

$$L_3 = -Z_y \sin \theta_i. \quad (11e)$$

From the expression of  $h_p^i$ , Eq. (I.19d), we get

$$M_1 = -K_2 \cos \theta_i, \quad (11f)$$

$$M_2 = K_1 \cos \theta_i - K_3 \sin \theta_i, \quad (11g)$$

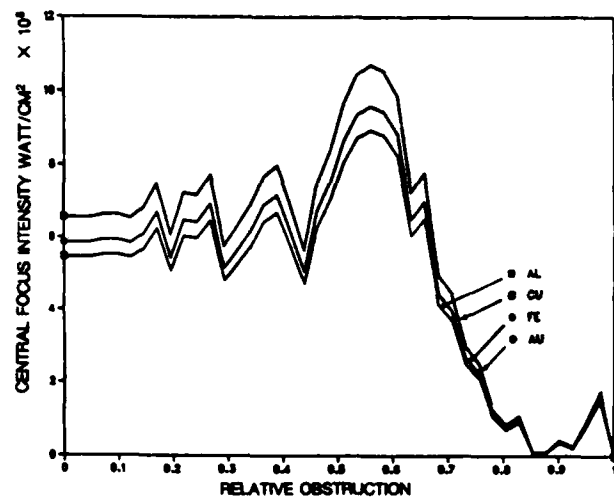


Fig. 4. Effect of the relative central obstruction of the paraboloidal mirror on the focal intensity ( $x_p = 0$ ,  $y_p = 0$ ,  $z_p = R$ ) for four metals.

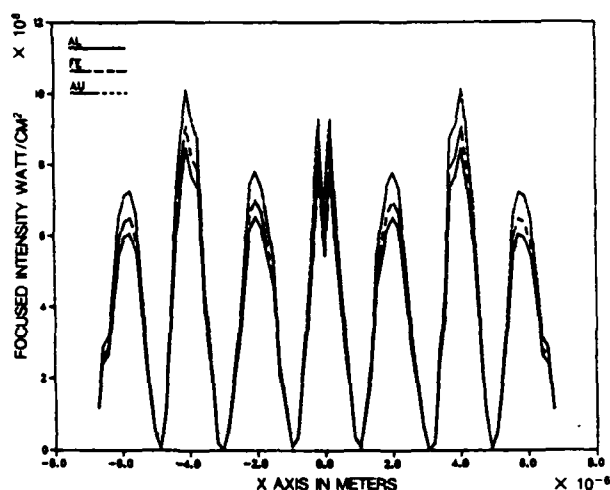


Fig. 5. Intensity distribution along the  $x$  axis in the focal plane for the four metals.

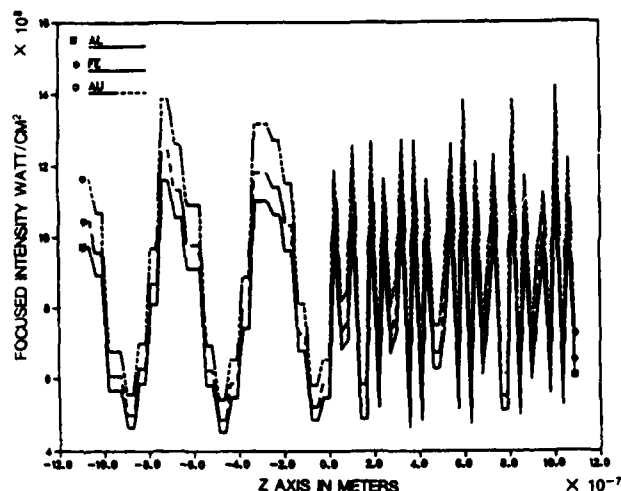


Fig. 6. Intensity distribution along the  $z$  axis in the focal volume for the four metals.  $z = 0$  in this figure corresponds to the focal point in Fig. 1.

and

$$M_3 = K_2 \sin \theta_i \quad (11h)$$

From the expression of  $e_p^i$ , Eq. (I.19c), we get

$$K_1 = -\hat{a} \cdot \hat{n} Z_x + \hat{a} \cdot \hat{\rho} [\sin \theta_i (Z_y^2 + 1) + Z_x \cos \theta_i] / C \sin \theta_{in} \quad (11i)$$

$$K_2 = -\hat{a} \cdot \hat{n} Z_y + \hat{a} \cdot \hat{\rho} (-Z_x Z_y \sin \theta_i + Z_y \cos \theta_i) / C \sin \theta_{in} \quad (11j)$$

and

$$K_3 = \hat{a} \cdot \hat{n} + \hat{a} \cdot \hat{\rho} [(Z_x^2 + Z_y^2) \cos \theta_i + Z_x \sin \theta_i] / C \sin \theta_{in} \quad (11k)$$

Substituting the surface derivatives from Eqs. (5) and the vectorial products from Eqs. (7) into Eqs. (10) and (11), we get expressions for the  $S$ 's and the  $T$ 's in terms of the surface coordinates. Then the surface currents that contribute to

the reflected fields at  $P$  are given by Eqs. (II.20). The three components of the reflected electric field at  $P(x_p, y_p, z_p)$  are obtained by substituting the surface currents into Eq. (I.30). The results are

$$E_{sx}(P) = -[ik \exp(-ikD_0)/4\pi D_0] \iint_{\Omega} [E_i(Z)/D^2] \cos^2 \theta_{in} \\ \times \{[(y - y_p)S_3 - (Z - z_p)S_2]D - [(x - x_p) \\ \times (Z - z_p)T_3 - [(y - y_p)^2 + (Z - z_p)^2]T_1 \\ + (x - x_p)(y - y_p)T_2]\} \exp(ik\Phi) dx dy, \quad (12a)$$

$$E_{sy}(P) = -[ik \exp(-ikD_0)/4\pi D_0] \iint_{\Omega} [E_i(Z)/D^2] \cos^2 \theta_{in} \\ \times \{[(Z - z_p)S_1 - (x - x_p)S_3]D - [(x - x_p) \\ \times (y - y_p)T_1 - [(x - x_p)^2 + (Z - z_p)^2]T_2 \\ + (y - y_p)(Z - z_p)T_3]\} \exp(ik\Phi), \quad (12b)$$

and

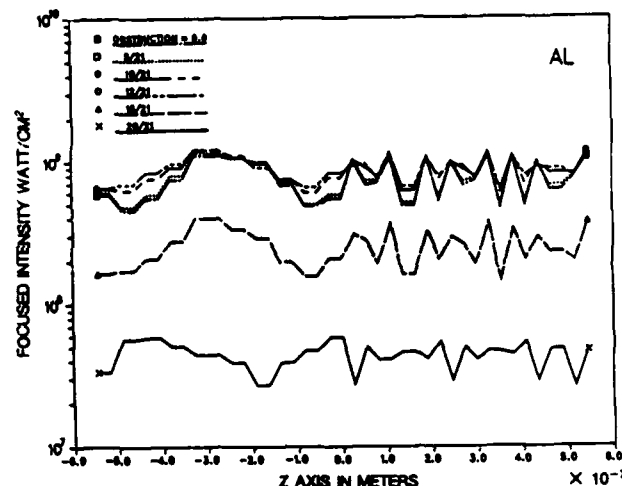


Fig. 7. Effect of the relative obstruction on the  $z$ -axial intensity distribution for an aluminum mirror.  $z = 0$  in this figure corresponds to the focal point in Fig. 1.

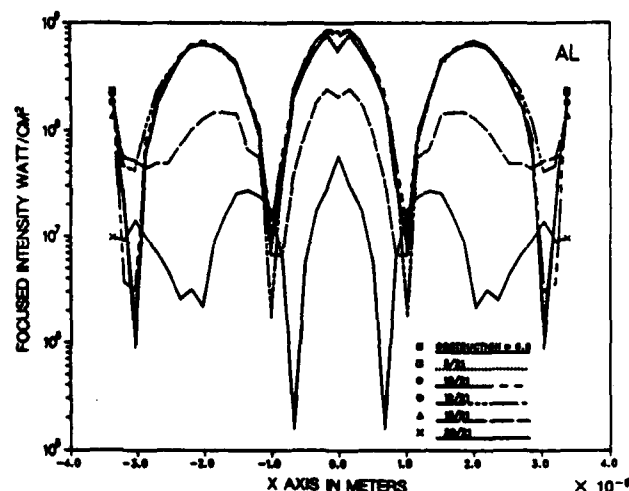


Fig. 8. Effect of the relative obstruction on the  $x$ -axial intensity distributions for an aluminum mirror.

